John Foundation Journal of EduSpark ISSN 2581-8414 (Print)
International Journal of Multidisciplinary Research Studies ISSN 2582-2128 (Online)

DUPLICATION SELF VERTEX SWITCHING OF Pm x Pn

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Abstract

A vertex $v \in V(G)$ is said to be a self vertex switching of G if G is isomorphic to G^v, where G^v is the graph obtained from G by deleting all edges of G incident to v and adding all edges incident to v which are not in G. Duplication of a vertex v of graph G produces a new graph G' by adding a new vertex v' such that $N(v') = N(v)$. In other words a vertex v' is said to be duplication of v if all the vertices which are adjacent to v in G are also adjacent to v' in G'. A vertex v is called a duplication self vertex switching of a graph G if the resultant graph obtained after duplication of v has v as a self vertex switching. In this paper, we find duplication self vertex switchings of P_m x P_n.

Keywords: *switching, self vertex switching, duplication self vertex switching, $dss_1(G)$.*

AMS Subject Classification: 05C60.

Introduction

For a finite undirected graph $G(V,E)$ with $|V| = p$ and a set $\sigma \subseteq V$, Seidel [9] defined the *switching* of G by σ as the graph $G^\sigma(V,E')$, which is obtained from G by removing all edges between σ and its compliment $V - \sigma$ and adding as edges all non edges between σ and $V-\sigma$. When $\sigma = \{v\} \subseteq V$, the corresponding switching $G^{\{v\}}$ is called a *vertex switching* and is denoted by G^v [10]. Switching is an equivalence relation and the associated equivalence classes are called *switching classes*. For a survey of switching classes of graphs we refer to Seidel [9]. A subset σ of $V(G)$ to be a *self switching* of G if G *isomorphic to* G^σ. The set of all self switchings of G with cardinality k is denoted by $SS_k(G)$ and its cardinality by $ss_k(G)$. If $k = 1$, then we call the corresponding self switching as *self vertex switching* [12]. Duplication of a vertex v of graph G produces a new graph G' by adding a new vertex v' such that $N(v') = N(v)$. In other words a vertex v' is said to be duplication of v if

ISSN 2581-8414 (Print)
ISSN 2582-2128 (Online)

all the vertices which are adjacent to v in G are also adjacent to v' in G'. The concept of duplication self vertex switching was introduced by C. Jayasekaran and V. Prabavathy [5]. A vertex v is called a *duplication self vertex switching* of a graph G if the resultant graph obtained after duplication of v has v as a self vertex switching. The number of duplication self vertex switching is denoted by $dss_1(G)$. For any vertex $v \in V(G)$, the *open neighbourhood* $N(v)$ of v is the set of all vertices adjacent to v. That is $N(v) = \{u \in V(G)/ uv \in E(G)\}$. The *closed neighbourhood* of v is defined by $N[v] = N(v) \cup \{v\}$. The ladder graph L_n on n vertices is defined as the Cartesian product of P_n and P_2. Two vertices u and v in G are said to be *interchange similar* if there is an automorphism α of G such that $\alpha(u) = v$ and $\alpha(v) = u$ [8]. In [11], a characterization is given for a cut vertex in G to be a self vertex switching where G is a connected graph such that any two self vertex switchings if exists, are interchange similar. The existance of graphs with given number of self vertex switchings were discussed in [1]. The trees [3] and unicyclic graphs [2], [4] are characterized for self vertex switchings. We consider simple graphs only.

We consider the following results which are required in the subsequent section.

Theorem 1.1. [5] If v is a duplication self vertex switching of a graph G of order p then p is even and $d_G(v) = p/2$.

Theorem 1.2. [5] Let G be a graph and let v be any vertex of G. Then v is a duplication self vertex switching of G iff there exists an automorphism on G which maps elements of $N(v)$ onto elements of $[N(v)]^c$.

Theorem 1.3. [6] If $dss_1(G) > 0$ then the number of duplication self vertex switchings in G is even.

Main results

Theorem 2.1.

$$dss_1(P_1 \times P_n) = \begin{cases} 2 \text{ if } n = 2 \text{ and } 4 \\ 0 \text{ otherwise} \end{cases}$$

Proof.

To prove this theorem, we need the following three cases.

Case (i). n=2

In this case, the graph $P_1 \times P_n$ is P_2. Since each vertex of P_2 is a duplication self vertex switching, $dss_1(P_1 \times P_n) = 2$ when n = 2.

Case (ii). n=4

In this case, the graph $P_1 \times P_n$ becomes P_4 and there are only two vertices of P_4 are of degree 2 and these two vertices are duplication self vertex switchings and hence $dss_1(P_1 \times P_n) = 2$ when n = 4.

Case (iii). $n \neq 2$ **and** 4

When n=1 or n = 3, the graph $P_1 \times P_n$ becomes P_1 or P_3. Since graphs with even number of vertices only can have duplication self vertex switching, $dss_1(P_1 \times P_n) = 0$ when n = 1 & 3. Also since $dss_1(P_n) = 0$ when $n \geq 5$, $dss_1(P_1 \times P_n) = 0$ when $n \geq 5$.

The theorem follows from Case(i), Case(ii) & Case(iii).

Theorem 2.2.

$$dss_1(P_2 \times P_n) = \begin{cases} 2 \text{ if } n = 1,3 \\ 4 \text{ if } n = 2 \\ 0 \text{ otherwise} \end{cases}$$

Proof.

To Prove his theorem, we need the following cases.

Case (i). n=1

Since $P_2 \times P_1 = P_1 \times P_2$, by case (i) of Theorem 2.1, it is clear that $dss_1(P_2 \times P_n) = 2$, when n=1.

Case (ii). n=2

In this case, the graph $P_2 \times P_n$ becomes the cycle C_4 and eah vetex of C_4 is the duplication self vertex switching and hence $dss_1 (P_2 \times P_n) = 4$ when n = 2.

Case (iii). n=3

When n=3, the graph $P_2 \times P_n$ is the ladder graph L_3 and there are only two vertices of L_3 are of degree 3 and these two vertices are duplication self vertex switchings of L_3 and hence $dss_1 (P_2 \times P_n) = 2$ when n = 3.

Case (iv). n $\geq$ **4**

When $n \geq 4$, the graph $P_2 \times P_n$ is a Ladder graph with 2n vertices and each vertex is of degree less than n and so $dss_1 (P_2 \times P_n) = 0$ when $n \geq 4$.

The theorem follows from Case(i), Case(ii), Case(iii) & Case(iv).

Theorem 2.3.

$$dss_1(P_3 \times P_n) = \begin{cases} 2 \text{ if } n = 2 \\ 0 \text{ otherwise} \end{cases}$$

Proof.

Case (i). n = 2

Since P_3 x P_2 = P_2 x P_3, by case(iii) of Theorem 2.2, it is clear that $dss_1(P_3 \text{ x } P_n) = 2$ when n = 2.

Case (ii). $\mathbf{n \neq 2}$

When n =1&3 the graph P_3 x P_n contains odd number of vertices and hence $dss_1(P_3 \text{ x } P_n) = 0$ when n = 1&3. When n > 3, graph P_3 x P_n is a graph of degree 3n and each vertex is of degree less than 3n/2 and hence $dss_1(P_3 \text{ x } P_n)$ = 0 when n > 3.

From Case(i) & Case(ii) the theorem is true.

Theorem 2.4. $dss_1(P_m \text{ x } P_n) = 0$ if m ≥ 4 and $n > 1$ or n ≥ 4 and $m > 1$.

If m ≥ 4 and $n > 1$, then the graph P_m x P_n is of order ≥ 4n and each vertex of P_m x P_n is of degree less than 4n/2 or and hence $dss_1(P_m \text{ x } P_n) = 0$ when m ≥ 4 and $n > 1$. Similarly we can prove that $dss_1(P_m \text{ x } P_n) = 0$ when n ≥ 4 and $m > 1$.

Conclusion

In this paper, we have proved that the number of duplication self vertex switching of P_m x P_n is zero if if m ≥ 4 and $n > 1$ or n ≥ 4 and $m > 1$. Also we found the number of duplication self vertex switching of P_m x P_n when m ≤ 4 or $n \leq 4$ and $m, n \geq 1$.

References

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[1] Jayasekaran, C. Graphs with a given numn and eachber of self vertex switchings, *International Journal of Algorithms, Computing and Mathematics*, vol. 3(2010), no. 3, pp 27-36.

[2] Jayasekaran, C. Self vertex switchings of connected unicyclic graphs, *Journal of Discrete Mathematical Sciences and Crypto graphy*, vol. 15(2012), no. 6, pp. 377-388.

[3] Jayasekaran, C. Self vertex swit chings of trees, *Ars Combinatoria*, 127(2016), pp. 33-43.

[4] Jayasekaran, C. Self vertex switchings of disconnected uni cyclic graphs, *Ars Combinatoria*, 129(2016), pp. 51-63.

[5] Jayasekaran, C. & Prabavathy, V. A characterisation of duplication self vertex switching in graphs, *International Journal of Pure and*

ISSN 2581-8414 (Print)
ISSN 2582-2128 (Online)

Applied Mathematics, vol. 118, (2017), no. 2.

[6] Jayasekaran, C. & Prabavathy, V. Duplication self vertex switching in some graphs, (Communicated).

[7] Jayasekaran, C. & Prabavathy, V. Some results on duplication self vertex switchings, *International Journal of Pure and Applied Mathematics*, vol. 116(2017), no. 2, pp. 427-435.

[8] Lauri, J. Pseudosimilarity in graphs - a survey, *Ars Comb inatoria*, 46(1997), pp. 77-95.

[9] Seidel, J., J. A survey of two graphs, in *Proceedings of the Inter National Coll. Theorie combina torie* (Rome 1973), Tomo I, Acca. Naz. Lincei, (1976), pp. 481- 511.

[10] Stanley, R. Reconstruction from vertex switching, *J. Combi. Theory. Series B*, vol. 38 (1985), pp. 38-142.

[11] Vilfred, V., Paulraj Joseph, J. & Jeyasekaran, C. Branches and Joints in the study of self switching of graphs, *The Journal of Combinatorial Mathematics and combinatorial Computing*, 67 (2008), pp. 111-122.

[12] Vilfred, V. & Jayasekaran, C. Interchange similar self vertex switchings in graphs, *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 12(2009), no. 4, pp. 467- 480.

To cite this article

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Prabavathy, V. (2022). Duplication Self Vertex Switching of Pm x Pn. *John Foundation Journal of EduSpark, 4*(3), 34-38.

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